Bulletin of the Seismological Society of America, Vol. 86, No. 4, pp. 1091–1106, August 1996

Simulating Seismic Wave Propagation in 3D Elastic Media Using Staggered-Grid Finite Differences

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Abstract This article provides an overview of the application of the staggered-grid finite-difference technique to model wave propagation problems in 3D elastic media. In addition to presenting generalized, discrete representations of the differential equations of motion using the staggered-grid approach, we also provide detailed formulations that describe the incorporation of moment-tensor sources, the implementation of a stable and accurate representation of a planar free-surface boundary for 3D models, and the derivation and implementation of an approximate technique to model spatially variable anelastic attenuation within time-domain finite-difference computations. The comparison of results obtained using the staggered-grid technique with those obtained using a frequency-wavenumber algorithm shows excellent agreement between the two methods for a variety of models. In addition, this article also introduces a memory optimization procedure that allows large-scale 3D finite-difference problems to be computed on a conventional, single-processor desktop workstation. With this technique, model storage is accommodated using both external (hard-disk) and internal (core) memory. To reduce system overhead, a cascaded time update procedure is utilized to maximize the number of computations performed between I/O operations. This formulation greatly expands the applicability of the 3D finite-difference technique by providing an efficient and practical algorithm for implementation on commonly available workstation platforms.

Introduction

Modern computational efficiency has advanced to a state where we can begin to calculate wave-field simulations for realistic 3D models at frequencies of interest to both seismologists and engineers. The most general of these numerical methods are grid-based techniques that track the wave field on a dense 3D grid of points, e.g., the finite-difference (FD), finite-element (FE), and pseudospectral (PS) methods. Various algorithms have been developed to implement these techniques, and while there will always be debate as to which one is the "best" technique, each method has its merits and pitfalls.

Our approach uses a staggered-grid finite-difference algorithm to model the first-order elastodynamic equations of motion expressed in terms of velocity and stress. In seismic applications, the velocity-stress formulation was first used by Madariaga (1976) to model fault-rupture dynamics. Virieux (1984, 1986) and Levander (1988) have since extended the technique to model seismic wave propagation in 2D media, and the formulation for 3D media is outlined by Randall (1989) and Yomogida and Etgen (1993). The advantages of the staggered-grid formulation are (1) source insertion is straightforward and can be expressed in terms of velocity (via body forces) or stress; (2) a stable and accurate representation for a planar free-surface boundary is easily implemented; (3) since the finite-difference operators are local, the entire model does not have to reside in core memory all at once; (4) it is easily extended to high-order spatial difference operators; (5) the method can be interfaced with other modeling techniques by expressing the input wave field along a boundary of the finite-difference grid; and (6) the algorithm is easily implemented on scalar, vector, or parallel computers.

In the following sections, we outline the numerical approach beginning with the equations of motion and then describe their discrete formulation using the staggered-grid approach. We do not provide a detailed analysis of the development of absorbing boundary conditions, stability of the numerical system, or issues regarding numerical grid dispersion, as these topics are all adequately covered in the before-mentioned articles. We do, however, discuss in detail some new ideas related to incorporating earthquake (double-couple) sources, free-surface boundary implementation, and modeling spatially variable anelasticity (using Q). In addition, we describe a memory optimization technique, which allows the computation of large-scale 3D finite-difference problems using only a single-processor desktop workstation.